Two-Port Parameters

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \text{S} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \text{S} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \text{S} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \text{S}$$

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \Omega \qquad a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \text{S} \qquad a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad b_{12} = -\left.\frac{V_2}{I_1}\right|_{V_1=0} \Omega \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0} \text{S} \qquad b_{22} = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

Two-Port Parameters

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \Omega \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \text{S}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \text{S} \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \Omega$$

General Equations

$$V_1 = z_{11}I_1 + z_{12}I_2 \qquad I_1 = y_{11}V_1 + y_{12}V_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2 \qquad I_2 = y_{21}V_1 + y_{22}V_2$$
$$V_1 = a_{11}V_2 - a_{12}I_2 \qquad V_2 = b_{11}V_1 - b_{12}I_1$$
$$I_1 = a_{21}V_2 - a_{22}I_2 \qquad I_2 = b_{21}V_1 - b_{22}I_1$$
$$V_1 = h_{11}I_1 + h_{12}V_2 \qquad I_1 = g_{11}V_1 + g_{12}I_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2 \qquad V_2 = g_{21}V_1 + g_{22}I_2$$

TABLE 18.1 Parameter Conversion Table

$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$	$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$	$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$	$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$
$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$	$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$	$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$	$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$
$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$	$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$	$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$	$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$
$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$	$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$		

Copyright © 2011 Pearson Education, Inc. publishing as Prentice Hall

TABLE 18.1 Parameter Conversion Table

$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$	$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$	$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$	$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$
$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$	$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$	$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$	$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$
$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$	$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$		

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$
$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$
$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$
$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$
$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$
$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

Copyright © 2011 Pearson Education, Inc. publishing as Prentice Hall

Relationships for a reciprocal two-port circuit

$$z_{12} = z_{21} \qquad a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1 \qquad h_{12} = -h_{21}$$
$$y_{12} = y_{21} \qquad b_{11}b_{22} - b_{12}b_{21} = \Delta b = 1 \qquad g_{12} = -g_{21}$$

Relationships for a symmetric two-port circuit

$$z_{11} = z_{22} \qquad h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1 \qquad a_{11} = a_{22}$$
$$y_{11} = y_{22} \qquad g_{11}g_{22} - g_{12}g_{21} = \Delta g = 1 \qquad b_{11} = b_{22}$$

TABLE 18.2 Terminated Two-Port Equations

z Parameters	*y* Parameters
$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$	$Y_{in} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$
$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta yZ_gZ_L}$
$V_{Th} = \frac{z_{21}}{z_{11} + Z_g}V_g$	$V_{Th} = \frac{-y_{21}V_g}{y_{22} + \Delta yZ_g}$
$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$	$Z_{Th} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta yZ_g}$
$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$	$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta yZ_L}$
$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$	$\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$
$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$

Copyright © 2011 Pearson Education, Inc. publishing as Prentice Hall

TABLE 18.2 Terminated Two-Port Equations

a Parameters	*b* Parameters
$Z_{in} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$	$Z_{in} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$
$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$	$I_2 = \frac{-V_g\Delta b}{b_{11}Z_g + b_{21}Z_gZ_L + b_{22}Z_L + b_{12}}$
$V_{Th} = \frac{V_g}{a_{11} + a_{21}Z_g}$	$V_{Th} = \frac{V_g\Delta b}{b_{22} + b_{21}Z_g}$
$Z_{Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$	$Z_{Th} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$
$\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$	$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$
$\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$	$\frac{V_2}{V_1} = \frac{\Delta bZ_L}{b_{12} + b_{22}Z_L}$
$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$	$\frac{V_2}{V_g} = \frac{\Delta bZ_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_gZ_L}$

Copyright © 2011 Pearson Education, Inc. publishing as Prentice Hall

Relationships for two cascaded two-port circuits

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} \qquad a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} \qquad a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} \qquad a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22}$$